Calculus Warm Up #4-5

For: $s(t) = 2t^2 - 5$

1. Find the average rate of change on the interval [-3, 2]

$$\frac{s(-3) - s(2)}{-3-2}$$

-2

2. Find the instantaneous rate of change at each endpoint of the interval.

$s'(t) = 4t$

$s'(-3) = -12$

$s'(2) = 8$

Staple and turn in
Week 4 Classwork:

Warm up on top

(with new team number)

3.7 Related Rates WS

HW Questions: p. 152

In Exercises 37–42, find the equation of the tangent line and the normal line to the graph of the given equation at the indicated point.

39. $x^2 + y^2 = 20$ (2, 4)

$2x + 2y\frac{dy}{dx} = 0$

$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$

$m = \frac{-2}{4}$

$m = -\frac{1}{2}$

$\boxed{y - 4 = -\frac{1}{2}(x - 2)}$

normal line

$m = 2$

43. Find the points on the graph

(x, y)

$$f(x) = \frac{1}{3}x^3 + x^2 - x - 1$$

at which the slope is (a) -1,

$f'(x) = x^2 + 2x - 1$

$-1 = x^2 + 2x - 1$

In Exercises 45–48, find the derivative of the given function by using the definition of the derivative.

45. $f(x) = \frac{1}{x^2}$

In Exercises 49 and 50, derive the equations for the velocity and acceleration of a particle having the given position function.

49. $s = t + \frac{1}{t+1}$

$(t+1)^{-1}$

$v(t) = 1 - (t+1)^{-2}(1)$

$a(t) = -(-2)(t+1)^{-3}(1)$

$a(t) = \frac{2}{(t+1)^3}$

59. A point moves along the curve $y = \sqrt{x}$ in such a way that the y-value is increasing at the rate of 2 units per second. At what rate is x changing for the following values?

(a) $x = \frac{1}{2}$ (b) $x = 1$ (c) $x = 4$

$\rightarrow \frac{dy}{dt} = 2$

find $\frac{dx}{dt}$

$y = x^{1/2}$

$\frac{dy}{dt} = \frac{1}{2} x^{-1/2} \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{1}{2\sqrt{x}} \frac{dx}{dt}$

HW Questions: review ws

1. Use the definition of the derivative to find $f'(x)$ when $f(x)=2x^2+3$.

$$f'(x)=\lim_{\Delta x\to 0}\frac{2(x+\Delta x)^2+3-2x^2-3}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left(2x^2+4x\Delta x+(\Delta x)^2+3-2x^2-3\right)$$

$$=\lim_{\Delta x\to 0}\frac{1}{\cancel{\Delta x}}\left(\cancel{\Delta x}(4x+\Delta x)\right)=\boxed{4x}$$

2. An automobile's velocity starting from rest is given by $v(t)=\frac{100t}{2t+15}$ ft/sec.
Find the acceleration at t=10 seconds.

$$a(t)=v'(t)=[(2t+15)(100)-(100t)(2)\ /\ (2t+15)^2$$

$$a(t)=\frac{1500}{(2t+15)^2}$$

$$a(10)=\frac{1500}{(20+15)^2}=\frac{60}{49}\approx 1.224\ \text{ft/sec}^2$$

3. Determine the points (if any) at which $f(x)=2x^3-6x^2-18x+5$ has a horizontal tangent line. **(Show your work - graphing calculator solution alone is not acceptable).**

$$f'(x)=6x^2-12x-18$$
$$0=6(x^2-2x-3)$$
$$0=(x-3)(x+1)$$
$$x=3,-1$$

$$f(3)=2(27)-6(9)-18(3)+5$$
$$=54-54-54+5$$
$$=-49$$
$$f(-1)=-2-6+18+5$$
$$=15$$

$$\boxed{(3,-49)\quad(-1,15)}$$

4. Find the derivative of $f(x)=(x-1)(x^2-3x+2)$.

$$f'(x)=(x-1)(2x-3)+(1)(x^2-3x+2)\qquad (x-2)(x-1)$$
$$=2x^2-5x+3+x^2-3x+2$$
$$=\boxed{3x^2-8x+5}$$
$$=\boxed{(3x-5)(x-1)}$$

5. Find the derivative of $f(x) = \dfrac{x^3 + 3x + 2}{x^2 - 1}$.

$$f'(x) = \frac{(x^2-1)(3x^2+3) - (x^3+3x+2)(2x)}{(x^2-1)^2}$$

$$= (3x^4 - 3 - 2x^4 - 6x^2 - 4x)/(x^2-1)^2$$

$$= \frac{x^4 - 6x^2 - 4x - 3}{(x^2-1)^2}$$

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & -6 & -4 & -3 \\ & & -1 & 1 & 5 & -1 \\ \hline & 1 & -1 & -5 & 1 & \end{array}$$

doesn't have factors of $(x+1)$ or $(x-1)$

6. Find an equation of the tangent line to the graph of $f(x) = \dfrac{x-1}{x+1}$ at the point $(-2,3)$.

$$f'(x) = [(x+1)(1) - (x-1)(1)]/(x+1)^2$$

$$= \frac{2}{(x+1)^2}$$

$$f'(-2) = \frac{2}{(-2+1)^2} = 2$$

$$\boxed{y - 3 = 2(x+2)}$$

$$y = 2x + 7$$

7. Find $\dfrac{dy}{dx}$ given $y = \dfrac{1}{(x^2 - 3x)^2}$. $= (x^2 - 3x)^{-2}$

$$\frac{dy}{dx} = -2(x^2 - 3x)^{-3}(2x - 3)$$

$$= \frac{-2(2x-3)}{(x^2-3x)^3} = \frac{-4x+6}{(x^2-3x)^3}$$

8. Find the second derivative of $f(x)=\dfrac{1}{(2x-2)^3}$. $=(2x-2)^{-3}$

$f'(x)=-3(2x-2)^{-4}(2)$

$=-6(2x-2)^{-4}$

$f''(x)=24(2x-2)^{-5}(2)=\dfrac{48}{(2x-2)^5}$

9. Find $\dfrac{dy}{dx}$ by implicit differentiation for $3x^2y+2y^2x=-2$.

$3x^2\dfrac{dy}{dx}+6xy+2y^2(1)+4y\dfrac{dy}{dx}x=0$

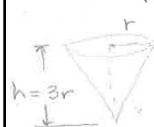
$\dfrac{dy}{dx}=\dfrac{-2y^2-6xy}{3x^2+4xy}$

10. Find the rate of change of the volume of a cone with height three times its radius, and radius increasing at 2 inches per minute, when a) the radius is 6 inches and b) the radius is 24 inches.

(Volume of a cone: $V=\frac{1}{3}\pi r^2h$)

Given: $\dfrac{dr}{dt}=2$ inches / min

find $\dfrac{dV}{dt}$ a) $r=6$ in b) $r=24$ in



$V=\frac{1}{3}\pi r^2(3r)$

$V=\pi r^3$

$\dfrac{dV}{dt}=3\pi r^2\dfrac{dr}{dt}$

a) $r=6$ in

$\dfrac{dV}{dt}=3\pi(6)^2(2)$

$\dfrac{dV}{dt}=216\pi$ in³/min ≈ 678.584

b) $r=24$ in

$\dfrac{dV}{dt}=3\pi(24)^2(2)$

$\dfrac{dV}{dt}=3456\pi$ in³/min. ≈ 10857.344

Chapter 4 Applications of Differentiation

4.1 Extrema:

Relative extrema and absolute extrema

Critical Numbers

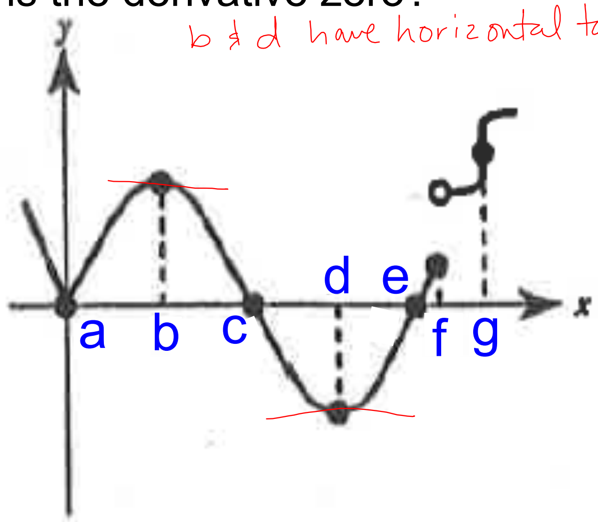
Where is the function not differentiable?

f (gap) g (vertical tangent)

a (pointy place)

Where is the derivative zero?

b & d have horizontal tangents



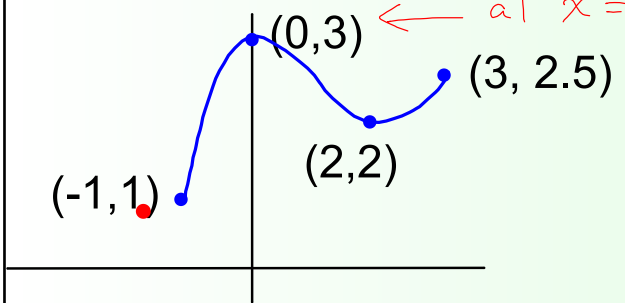
Extrema:

The extreme values of a function.
(outcomes or y-values)

Finding extrema on a closed interval: Looking for the highest and lowest points on that interval. The extrema will be the y-values there.

Closed interval: [-1, 3]

@ $x = -1$
Min value = 1



← at $x = 0$, maximum value = 3

Absolute Extrema

The highest or the lowest point on the interval including the endpoints

Ab. Max @ $x = 0$
Max Value = 3

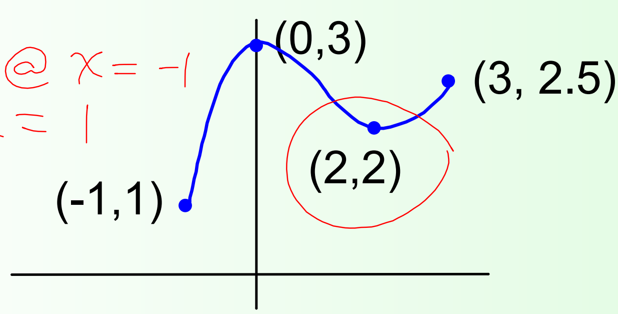
Ab. Min @ $x = -1$
Min Value = 1

Relative Extrema

← Not endpts.

Other places on the interval where the derivative is zero or undefined.

Relative min @ $x = 2$
Min Value is 2



Find the extrema of $f(x) = x^2 + 2$ on $[-1, 4]$

1) See where $f'(x) = 0$ or is undefined.

$f'(x) = 2x$

$0 = 2x$

$x = 0$

$f(0) = 2$

2) Check endpoints.

$f(-1) = 3$

$f(4) = 18$

Ab. Max @ $x = 4$

Max Value $= 18$

Ab Min @ $x = 0$

Min Value $= 2$

3) Compare all the outcomes, (y), for highest and lowest value.

Theorems and Definitions to understand:

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THEOREM 4.1
THE EXTREME VALUE THEOREM

If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.

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DEFINITION OF CRITICAL NUMBER

If f is defined at c, then c is called a **critical number** of f if $f'(c) = 0$ or if f' is undefined at c.

x-values where the derivative is zero or undefined.

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THEOREM 4.2
RELATIVE EXTREMA OCCUR ONLY AT CRITICAL NUMBERS

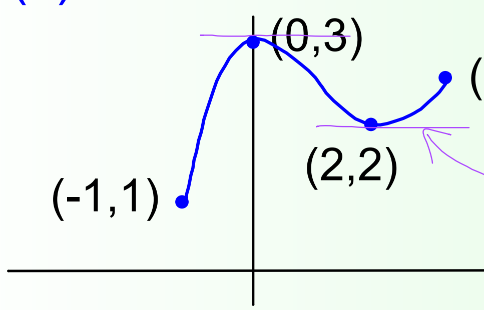
If f has a relative minimum or relative maximum at $x = c$, then c is a critical number of f.

so does not include end points

Critical Numbers of f,
(the x values where $f'(x) = 0$ or is undefined)

Critical #'s:
$x = 0, 2$

f(x):



horizontal tangent
$f'(x) = 0$

Find the extrema of $f(x) = 3x^4 - 4x^3$ on $[-1, 2]$

1) Find the critical numbers.
(Find x where $f'(x) = 0$ or is undefined.)

← f' is defined over entire domain

$f'(x) = 12x^3 - 12x^2$
$0 = 12x^2(x - 1)$
$x = 0, 1$ ← critical #'s

2) Evaluate f at critical numbers and endpoints.
(Find corresponding y-values.)

$f(0) = \underline{0}$ endpts: $f(-1) = \underline{7}$
$f(1) = \underline{-1}$ $f(2) = \underline{16}$

3) Compare all the outcomes, y, for highest and lowest value.

Min @ $x = 1$ Max @ $x = 2$
Min value $= -1$ Max value $= 16$

Which are absolute and which are relative?

Find the extrema of $f(x) = 2x - 3x^{2/3}$ on the interval $[-1, 3]$.

$f'(x) = 2 - \frac{2}{\sqrt[3]{x}}$

undefined @ $x = 0$

$0 = 2 - \frac{2}{\sqrt[3]{x}}$

$x = 1$

critical #'s

$x = 0, 1$

Evaluate @ critical #'s & endpts.

$f(0) = \underline{\underline{0}}$

$f(1) = -1$

$f(-1) = \underline{\underline{-5}}$

$f(3) \approx -0.24$

One more practice:

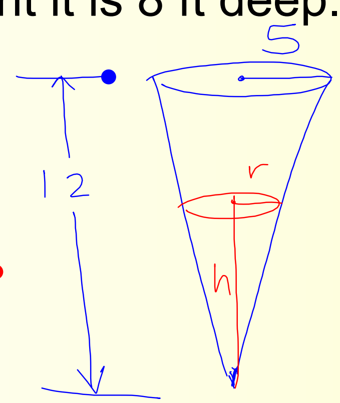
1. A conical tank (with vertex down) is 10 ft across the top and 12 ft deep. If water is flowing into the tank at a rate of 10 cubic ft per minute, find the rate of change of the depth of the water the instant it is 8 ft deep.

$\frac{r}{5} = \frac{h}{12}$

$r = \frac{5h}{12}$

$V = \frac{\pi}{3}\left(\frac{5h}{12}\right)^2 h$

then simplify before you differentiate.



$\frac{dh}{dt} = \frac{9}{10\pi}$ ft/min

Test Monday:

Differentiation with
- Power Rule
- Product Rule
- Quotient Rule
- Chain Rule

Implicit Differentiation

Particle Motion: Position, velocity, acceleration

Tangent Lines

Related Rates

HW: p. 160 # 1 - 25 odd